Information Flow Theory III
Cosmology in IFT

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1 Introduction

Understanding the dynamical origin of cosmological evolution remains one of the central challenges of modern theoretical physics. In the standard framework, the large-scale dynamics of the universe is described by general relativity coupled to a matter sector, supplemented by additional ingredients such as an inflaton field or a cosmological constant. While this approach is phenomenologically successful, it relies on the introduction of external structures whose fundamental origin remains unclear.

In parallel, the renormalization group (RG) has emerged as a powerful tool for describing the scale dependence of physical systems. In quantum field theory, and in particular in approaches based on the functional renormalization group (FRG) [3, 6, 4], the RG flow encodes how effective dynamics evolves with scale. This naturally raises the question of whether cosmological evolution itself can be understood as a manifestation of RG flow.

In this work, we address this question within the framework of Information Flow Theory (IFT). The foundations of the theory were established in [1], where a minimal covariant framework was introduced based on a scalar field interpreted as a local information variable. In this setting, spacetime geometry and physical interactions arise from the structure of scalar configurations. In [2], this framework was extended to include emergent geometry, fermionic degrees of freedom, and gauge interactions, leading to a structurally closed description in which the Standard Model gauge structure appears as a minimal and dynamically selected configuration.

The present work (IFT III) develops the cosmological implications of this framework. The central idea is that the RG scale is not an external parameter but is dynamically determined by the spectral properties of fluctuations on the emergent geometry. This leads to a relation of the form:

$$k = k[g]$$

which, in homogeneous and isotropic settings, reduces to a dependence on the Hubble parameter. As a consequence, the effective gravitational couplings become scale-dependent functions of the cosmological evolution:

$$G = G(H), \quad \Lambda = \Lambda(H)$$

This dynamical identification induces modifications of the gravitational field equations, leading to a coupled system in which spacetime dynamics and RG flow are intrinsically linked. Within a controlled derivative expansion, this results in modified Friedmann and Raychaudhuri equations, together with generalized conservation laws reflecting energy exchange between matter and the RG-induced sector.

A key aspect of the present approach is that it does not rely on specifying a particular RG trajectory. Instead, we analyze the cosmological consequences of general structural properties of the flow, such as scaling behavior, saturation, and smoothness. This allows us to identify generic dynamical regimes without introducing additional ad hoc assumptions.

Within this framework, several results emerge:

- ultraviolet scaling regimes can lead to accelerated expansion without introducing an independent inflaton field,
- classical cosmological evolution is recovered when the running of the couplings becomes negligible,
- infrared behavior of the flow can generate late-time acceleration,
- and, importantly, regimes of flow saturation can induce dynamical instabilities that provide a potential mechanism for the growth of cosmological inhomogeneities.

These results suggest that key features of cosmology—such as inflation, dark energy, and the emergence of structure—may admit a unified interpretation in terms of RG flow dynamics.

The paper is organized as follows. In Section 2, we establish the relation between RG flow and physical scales through the spectral properties of the fluctuation operator. In Section 3, we derive the effective gravitational action and its dependence on the RG scale. Section 4 presents the variation of the scale-dependent action and the resulting modified field equations. In Section 5, these equations are specialized to a cosmological background, leading to modified Friedmann dynamics. Section 6 analyzes the resulting cosmological regimes, while Section 7 introduces the mechanism of flow saturation and the associated dynamical instabilities. We conclude with a discussion of the implications and limitations of the framework.

Throughout this work, we restrict attention to a controlled approximation based on the Einstein–Hilbert truncation and a derivative expansion of the RG scale. The corresponding technical derivations and validity conditions are presented in the appendices.

2 RG Flow and Physical Scales

2.1 Functional Renormalization Group Framework

The dynamics of the scalar field is governed by the effective average action $\Gamma_k[\lambda]$, which satisfies the functional renormalization group (FRG) equation [3, 4, 5]:

$$\partial_k \Gamma_k = \frac{1}{2} \mathrm{Tr} \left[(\Gamma_k^{(2)} + R_k)^{-1} \partial_k R_k \right]$$

Here, $\Gamma_k^{(2)}$ denotes the second functional derivative with respect to the field, and R_k is an infrared regulator suppressing modes with eigenvalues below the scale k.

Proposition 2.1. *The operator $(\Gamma_k^{(2)} + R_k)^{-1}$ defines a scale-dependent propagator whose spectral properties control the effective degrees of freedom contributing to the flow at scale k.*

Proof. The regulator R_k adds a positive contribution to low eigenvalues, effectively suppressing modes with $\lambda \lesssim k^2$. As a result, the trace is dominated by modes with $\lambda \sim k^2$, which determine the effective dynamics at scale k. $\square$

2.2 Spectral Structural Functional

To quantify the effective number of active degrees of freedom, we introduce the spectral functional:

$$\mathcal{C}(k) = \mathrm{Tr} \left[(\Gamma_k^{(2)} + R_k)^{-1} \right]$$

Proposition 2.2. *The functional $\mathcal{C}(k)$ provides a measure of the density of modes contributing to the RG flow at scale k.*

Proof. Expanding in eigenvalues $\{\lambda_n(k)\}$ of $\Gamma_k^{(2)}$, we obtain:

$$\mathcal{C}(k) = \sum_n \frac{1}{\lambda_n(k) + R_k}$$

Since the regulator suppresses low modes and large eigenvalues are damped by the inverse, the dominant contribution arises from modes satisfying $\lambda_n(k) \sim k^2$. $\square$

A detailed analysis of the spectral density associated with scalar fluctuations is provided in Appendix Q.

2.3 Spectral Selection of the RG Scale

We now relate the RG scale k to physical scales in a cosmological background.

Theorem 2.3 (Spectral Selection Principle)**.** *Let $(\mathcal{M}, g_{\mu\nu})$ be a spacetime geometry. The dominant contribution to the RG flow arises from modes satisfying:*

$$\lambda_{\text{eff}} \sim k^2$$

where λ_{eff} denotes a characteristic spectral scale of the operator $\Gamma_k^{(2)}$.

Proof. The FRG trace is dominated by modes for which $\lambda_n + R_k$ is minimal. Since R_k suppresses modes with $\lambda \ll k^2$, the dominant contribution comes from modes satisfying $\lambda_n \sim k^2$, establishing the spectral matching condition. $\square$

Remark (Scheme Dependence). The definition of λ_{eff} and the associated RG scale is not unique and depends on the choice of regulator and spectral weighting. However, physical observables are invariant under reparametrizations of the RG scale (Appendix O).

2.4 Spectral Scale in FLRW Geometry

Consider a spatially homogeneous and isotropic spacetime with metric:

$$ds^2 = -dt^2 + a(t)^2 d\vec{x}^2$$

Proposition 2.4. *In a FLRW background, the characteristic spectral scale of second-order operators is determined by local curvature invariants and satisfies:*

$$\lambda_{\text{eff}}(t) \sim H(t)^2 + \dot{H}(t)$$

up to higher-derivative corrections.

Proof. The operator $\Gamma_k^{(2)}$ contains covariant derivatives and curvature contributions. In FLRW spacetime, curvature invariants scale as:

$$R \sim H^2 + \dot{H}$$

By standard spectral asymptotics for elliptic operators [11], the eigenvalue distribution is controlled by these invariants. A detailed derivation based on a local derivative expansion is given in Appendix P. $\square$

2.5 Dynamical Determination of the RG Scale

Combining spectral selection with the geometric estimate yields:

Theorem 2.5 (Dynamical RG Scale)**.** *In a homogeneous and isotropic cosmological background, the RG scale satisfies:*

$$k^2 \simeq H(t)^2 + \dot{H}(t)$$

at leading order in a derivative expansion.

Proof. The relation follows from identifying the RG scale with the characteristic spectral scale λ_{eff}, together with its expression in terms of curvature invariants derived above. □

Validity Regime. This identification holds within a controlled derivative expansion:

$$\frac{\dot{H}}{H^2} \ll 1, \quad \frac{\ddot{H}}{H^3} \ll 1,$$

where higher-order curvature and non-local corrections are suppressed (Appendix P).

Interpretation. The relation between k and geometric quantities should therefore be understood as an effective, scale-dependent identification valid within this approximation scheme, rather than an exact equality.

At leading order, this reduces to:

$$k \sim H(t)$$

2.6 Interpretation

The RG scale is not introduced as an external parameter but emerges dynamically from the spectral structure of the theory. Its identification with geometric quantities reflects the fact that the flow is governed by the spectrum of fluctuations on the background spacetime.

While the precise definition of $k[g]$ is scheme-dependent and not uniquely fixed beyond the derivative expansion, the physical content of the theory depends only on the RG trajectory in coupling space (Appendix O).

This establishes a robust — though approximation-dependent — link between renormalization group flow and cosmological evolution, which will be used in the following sections to derive the effective dynamics.

3 Effective Gravitational Dynamics

3.1 RG-Induced Effective Action

The functional renormalization group flow defines a scale-dependent effective action Γ_k. In the low-energy regime, and under standard assumptions of locality and diffeomorphism invariance, the effective action admits an expansion in local geometric invariants [6, 7, 8].

Proposition 3.1. *At leading order in a derivative expansion, the effective action takes the form:*

$$\Gamma_k[g] = \int d^4x \sqrt{-g} \left[\frac{1}{16\pi G(k)} R - \Lambda(k) \right] + \mathcal{O}(\nabla^4)$$

where $G(k)$ and $\Lambda(k)$ are scale-dependent couplings.

Sketch. General covariance restricts the action to scalar invariants constructed from the metric. At lowest order in derivatives, these are the Ricci scalar R and a constant term. Higher-order curvature invariants such as R^2 and $R_{\mu\nu}R^{\mu\nu}$ are suppressed in the infrared regime. A detailed derivation within the present framework is provided in Appendix B. □

Remark (Truncation). The Einstein–Hilbert form should be understood as the leading term of a systematic derivative expansion. Its validity relies on the suppression of higher-curvature operators, which is controlled in the regime where curvature invariants remain small compared to the RG scale (see Appendix P).

3.2 Dynamical Dependence on Geometry

From the spectral analysis of Section 2, the RG scale is a functional of the metric:

$$k = k[g_{\mu\nu}]$$

Proposition 3.2. *The effective couplings become functionals of the geometry:*

$$G = G(g_{\mu\nu}), \quad \Lambda = \Lambda(g_{\mu\nu})$$

Proof. The couplings $G(k)$ and $\Lambda(k)$ depend on the RG scale. Since k itself depends on the spectral properties of the metric, the result follows by composition. □

In a homogeneous and isotropic background, this dependence reduces to:

$$G = G(H), \quad \Lambda = \Lambda(H)$$

where H is the Hubble parameter. The closure of this dependence is established in Appendix J.

3.3 Scheme Dependence and Physical Content

Proposition 3.3. *The functional dependence $G = G[g]$ and $\Lambda = \Lambda[g]$ is not unique, but physical observables are invariant under reparametrizations of the RG scale.*

Proof. The RG scale admits redefinitions of the form $k \to \tilde{k} = f(k)$. Under such transformations, the running couplings are reparametrized accordingly, while their composition with $k[g]$ remains unchanged. The invariance of the field equations under these transformations is demonstrated in Appendix O. □

This ensures that the dynamical dependence of couplings on geometry is physically meaningful despite the non-uniqueness of the RG parametrization.

3.4 Absence of External Scale Identification

A key feature of the present framework is that the identification of the RG scale is not imposed externally.

Proposition 3.4. *The relations $G = G(H)$ and $\Lambda = \Lambda(H)$ arise dynamically from the spectral structure of the theory and do not rely on an external identification of the RG scale.*

Proof. The RG scale is determined by the dominant eigenvalues of the fluctuation operator $\Gamma_k^{(2)}$. As shown in Section 2, these eigenvalues are controlled by curvature invariants of the spacetime geometry. The relation $k \sim H$ follows as the leading-order term of a controlled derivative expansion (Appendix P), without introducing additional assumptions. □

3.5 Validity Regime

The effective action and its geometric interpretation are valid under the following conditions:

- the derivative expansion is controlled, i.e.

$$\frac{R}{k^2} \ll 1$$

  so that higher-order curvature terms are subleading (Appendix P),
- the RG flow defines smooth running couplings $G(k)$ and $\Lambda(k)$ in the regime considered,
- the spacetime geometry varies slowly on scales comparable to k^{-1}.

Proposition 3.5. *Under these conditions, the Einstein–Hilbert truncation provides a consistent approximation to the full effective action.*

3.6 Interpretation

The resulting framework describes an effective theory of gravity in which the couplings are dynamically determined rather than fundamental constants.

In particular:

- the gravitational coupling G encodes the scale-dependent response of the system to curvature,
- the cosmological term Λ reflects the effective vacuum energy induced by the RG flow,
- both quantities evolve with the geometry through the relation $k = k[g]$, defined by the spectral properties of fluctuations.

This establishes a direct and internally consistent connection between renormalization group flow and gravitational dynamics. The resulting field equations, obtained by varying the action, will be derived in the next section.

4 Variation of the Scale-Dependent Action

4.1 General Structure of the Variation

We consider the effective action:

$$\Gamma[g] = \int d^4x \sqrt{-g} \left[\frac{1}{16\pi G(k)} R - \Lambda(k) \right],$$

where the RG scale k depends on the metric through its spectral properties, as discussed in Section 2.

As a result, the couplings satisfy:

$$G = G(k[g]), \quad \Lambda = \Lambda(k[g]),$$

and are therefore, in general, non-local functionals of the metric.

Remark (Non-locality). The dependence $k[g]$ arises from the spectrum of the fluctuation operator $\Gamma_k^{(2)}$ and is therefore non-local in the exact theory. The local expressions used below are obtained within a controlled derivative expansion (Appendix P).

Proposition 4.1. *The variation of the effective action decomposes as:*

$$\delta\Gamma = \delta\Gamma_{\mathrm{EH}} + \delta\Gamma_{\mathrm{RG}},$$

where $\delta\Gamma_{\mathrm{EH}}$ is the standard Einstein–Hilbert variation with spacetime-dependent couplings, and $\delta\Gamma_{\mathrm{RG}}$ encodes the implicit dependence of $k[g]$.

Remark (RG scheme). The functional dependence $k[g]$ is not unique and is defined up to RG reparametrizations. Physical observables remain invariant under such transformations (Appendix O).

4.2 Variation of the Einstein–Hilbert Term

We consider:

$$\Gamma_{\mathrm{EH}} = \int d^4x\sqrt{-g}\,\frac{R}{16\pi G}.$$

Proposition 4.2. *The variation yields:*

$$\delta\Gamma_{\mathrm{EH}} = \int d^4x\sqrt{-g}\left[\frac{1}{16\pi G}G_{\mu\nu} + \frac{1}{16\pi}\left(\nabla_\mu\nabla_\nu G^{-1} - g_{\mu\nu}\Box G^{-1}\right)\right]\delta g^{\mu\nu}.$$

Proof. The result follows from the standard variation of the Einstein–Hilbert action together with the spacetime dependence of G. Integration by parts generates the derivative terms involving G^{-1}. Details are provided in Appendix D. □

4.3 Variation of the Cosmological Term

We consider:

$$\Gamma_\Lambda = -\int d^4x\sqrt{-g}\,\Lambda(k).$$

Proposition 4.3. *The variation is:*

$$\delta\Gamma_\Lambda = \int d^4x\sqrt{-g}\left[\frac{1}{2}\Lambda g_{\mu\nu} + \frac{d\Lambda}{dk}\frac{\delta k}{\delta g^{\mu\nu}}\right]\delta g^{\mu\nu}.$$

4.4 Variation of the RG Scale

The variation of the RG scale is formally given by:

$$\delta k(x) = \int d^4x'\,\frac{\delta k(x)}{\delta g_{\alpha\beta}(x')}\,\delta g_{\alpha\beta}(x').$$

Proposition 4.4. *The functional derivative $\delta k/\delta g_{\mu\nu}$ encodes non-local and higher-derivative contributions to the field equations.*

Proof. The RG scale depends on the spectrum of $\Gamma_k^{(2)}$, which involves curvature invariants and their derivatives. In the derivative expansion, this dependence can be organized as a local series (Appendix P). □

FLRW reduction. In a homogeneous and isotropic background, where $k \simeq k(H)$ (Section 2), one obtains:

$$\delta k \simeq \frac{dk}{dH}\,\delta H,$$

with δH involving derivatives of the metric.

4.5 Explicit Structure of $\Theta_{\mu\nu}$ in FLRW

We now compute explicitly the contribution arising from the variation of the RG scale in a homogeneous and isotropic spacetime.

Choice of congruence. We introduce a timelike unit vector field u^μ satisfying:

$$u^\mu u_\mu = -1.$$

In a FLRW spacetime, symmetry uniquely selects the comoving congruence associated with the cosmological fluid. In more general spacetimes, the construction may depend on the choice of observer.

Proposition 4.5. *In a FLRW spacetime, the tensor $\Theta_{\mu\nu}$ takes the form:*

$$\Theta_{\mu\nu} = -A(H)\,u_\mu u_\nu - B(H)\,H\,h_{\mu\nu},$$

where $h_{\mu\nu} = g_{\mu\nu} + u_\mu u_\nu$, and $A(H)$, $B(H)$ are functions determined by the RG flow.

Proof. Using $k = k(H)$, one has:

$$\delta k = \frac{dk}{dH}\delta H.$$

In a FLRW spacetime, the Hubble parameter can be written covariantly as:

$$H = \frac{1}{3}\nabla_\mu u^\mu.$$

Its variation yields:

$$\delta H = -\frac{1}{2}H\,u_\mu u_\nu \delta g^{\mu\nu} + \frac{1}{6}\nabla_\mu(u_\nu \delta g^{\mu\nu}).$$

Substituting into $\delta\Lambda(k[g])$ and integrating by parts, one obtains:

$$\Theta^{(\Lambda)}_{\mu\nu} \sim -\frac{d\Lambda}{dk}\frac{dk}{dH}\left[H u_\mu u_\nu + \nabla_{(\mu}u_{\nu)}\right].$$

A similar contribution arises from the variation of $G(k[g])$. In a FLRW spacetime, homogeneity and isotropy imply:

$$\nabla_{(\mu}u_{\nu)} = H h_{\mu\nu},$$

a relation which does not hold in general spacetimes.

This yields the stated form. □

Proposition 4.6. *The tensor $\Theta_{\mu\nu}$ can be written as an effective perfect fluid:*

$$\Theta_{\mu\nu} = (\rho_{RG} + p_{RG})u_\mu u_\nu + p_{RG} g_{\mu\nu},$$

with:

$$\rho_{RG}(H) = A(H), \quad p_{RG}(H) = B(H)\,H.$$

Covariant status. The above expression is valid in a FLRW background. A fully covariant expression of $\Theta_{\mu\nu}$ in a general spacetime would require a non-perturbative definition of $k[g]$.

4.6 Field Equations

Collecting all contributions, the field equations take the form:

$$\frac{1}{8\pi G}G_{\mu\nu} + \Lambda g_{\mu\nu} + \Delta^{\mathrm{RG}}_{\mu\nu} = T_{\mu\nu},$$

where:

$$\Delta^{\mathrm{RG}}_{\mu\nu} = \nabla_\mu\nabla_\nu G^{-1} - g_{\mu\nu}\Box G^{-1} + \Theta_{\mu\nu}.$$

4.7 Structure of the RG Corrections

Proposition 4.7. *The tensor $\Delta^{\mathrm{RG}}_{\mu\nu}$ decomposes into:*

- *geometric contributions from gradients of G,*
- *dynamical contributions encoded in $\Theta_{\mu\nu}$.*

4.8 Covariance and Conservation

Proposition 4.8. *The modified field equations satisfy:*

$$\nabla^\mu\left(\frac{1}{8\pi G}G_{\mu\nu} + \Lambda g_{\mu\nu} + \Delta^{\mathrm{RG}}_{\mu\nu}\right) = 0.$$

Proof. This follows from the variational origin of the action and the Bianchi identity. Details are provided in Appendix K. □

4.9 Validity and Limitations

The derivation relies on:

- the Einstein–Hilbert truncation,
- the derivative expansion of $k[g]$ (Appendix P),
- the use of FLRW symmetry in the explicit evaluation of $\Theta_{\mu\nu}$.

In particular, the explicit form of $\Theta_{\mu\nu}$ is not fully covariant and is restricted to homogeneous and isotropic backgrounds.

4.10 Interpretation

The resulting equations describe a modified gravitational dynamics in which:

- the gravitational coupling becomes spacetime-dependent,
- RG-induced terms generate effective fluid contributions,
- the tensor $\Theta_{\mu\nu}$ provides an explicit description of RG backreaction in cosmology within the symmetry-reduced setting.

In the next section, we specialize these equations to derive the modified Friedmann dynamics.

5 Modified Cosmological Equations

5.1 FLRW Background

We consider a spatially homogeneous and isotropic spacetime with metric:

$$ds^2 = -dt^2 + a(t)^2 d\vec{x}^2$$

where $a(t)$ is the scale factor and $H = \dot{a}/a$ is the Hubble parameter.

The matter content is modeled as a perfect fluid:

$$T_{\mu\nu} = (\rho + p)u_\mu u_\nu + p g_{\mu\nu}$$

5.2 Field Equations in Cosmology

The modified gravitational equations derived in Section 4 take the form:

$$\frac{1}{8\pi G} G_{\mu\nu} + \Lambda g_{\mu\nu} + \Delta^{\mathrm{RG}}_{\mu\nu} = T_{\mu\nu}$$

In a FLRW background, the RG scale reduces to $k \simeq k(H)$ at leading order in the derivative expansion (Section 2 and Appendix P), implying:

$$G = G(H), \quad \Lambda = \Lambda(H)$$

The closure of this dependence is established in Appendix J.

5.3 Modified Friedmann Equation

The (00) component of the Einstein tensor is:

$$G_{00} = 3H^2$$

The RG correction tensor contributes:

$$\Delta^{\mathrm{RG}}_{00} = -3H \frac{d}{dt} G^{-1} + \Theta_{00}$$

Proposition 5.1. *The cosmological evolution satisfies a modified Friedmann equation:*

$$H^2 = \frac{8\pi}{3} G(H)\rho + \frac{\Lambda(H)}{3} + \Delta_{RG},$$

where:

$$\Delta_{RG} = -\frac{1}{3} H \frac{d}{dt} \ln G(H) + \frac{1}{3}\Theta_{00}$$

Proof. Substituting the FLRW expressions into the field equations and isolating H^2 yields the result. A detailed derivation is provided in Appendix H. □

5.4 Modified Raychaudhuri Equation

The spatial components yield:

$$G_{ij} = -(2\dot{H} + 3H^2)g_{ij}$$

The RG correction contributes:

$$\Delta_{ij}^{\mathrm{RG}} = \left[\ddot{G}^{-1} + 3H\dot{G}^{-1}\right] g_{ij} + \Theta_{ij}$$

Proposition 5.2. *The evolution of the Hubble parameter satisfies:*

$$\dot{H} = -4\pi G(H)(\rho + p) + \Xi_{RG},$$

where:

$$\Xi_{RG} = \frac{1}{2}\left[\ddot{G}^{-1} + 3H\dot{G}^{-1}\right] + \Xi_\Theta$$

and Ξ_Θ collects the contributions from $\Theta_{\mu\nu}$.

Proof. Combining the spatial components with the modified Friedmann equation allows one to isolate $\dot{H}$. The detailed derivation is given in Appendix I. □

5.5 Structure of RG Corrections

The modified cosmological equations contain three distinct contributions:

- standard Einstein terms involving $G(H)$ and $\Lambda(H)$,
- derivative terms involving $\dot{G}$ and $\ddot{G}$,
- additional contributions encoded in $\Theta_{\mu\nu}$.

Proposition 5.3. *The RG corrections introduce higher-order time derivatives and encode effective non-locality through the dependence of $k[g]$ on the geometry.*

Proof. Since $G = G(H)$ and H depends on derivatives of the metric, time derivatives of G involve $\dot{H}$ and higher derivatives. Moreover, $\Theta_{\mu\nu}$ arises from the variation of $k[g]$, which is non-local at the fundamental level but admits a local derivative expansion (Appendix P). □

5.6 Consistency Check

In the limit of constant couplings:

$$G = \text{const}, \quad \Lambda = \text{const},$$

all RG corrections vanish:

$$\Delta_{RG} = 0, \quad \Xi_{RG} = 0,$$

and the standard Friedmann equations are recovered:

$$H^2 = \frac{8\pi G}{3}\rho + \frac{\Lambda}{3}, \quad \dot{H} = -4\pi G(\rho + p)$$

5.7 Validity Regime

The modified equations are valid under the assumptions:

- the derivative expansion is controlled:
$$\frac{\dot{H}}{H^2} \ll 1, \quad \frac{\ddot{H}}{H^3} \ll 1$$
(Appendix P),
- higher-curvature terms in the effective action are subleading,
- the RG flow defines smooth functions $G(H)$ and $\Lambda(H)$.

5.8 Interpretation

The resulting cosmological system describes a dynamical interplay between spacetime expansion and renormalization group flow.

In particular:

- the gravitational coupling evolves with the expansion rate,
- additional terms encode the backreaction of the RG flow,
- deviations from standard cosmology arise naturally in regimes where the running of couplings is significant.

These equations provide the starting point for the analysis of cosmological regimes in the following sections, including inflationary dynamics and the emergence of instabilities.

6 Energy Exchange and Conservation Laws

6.1 Covariant Divergence of the Field Equations

We consider the modified field equations:

$$\frac{1}{8\pi G} G_{\mu\nu} + \Lambda g_{\mu\nu} + \Delta^{\text{RG}}_{\mu\nu} = T_{\mu\nu}$$

Taking the covariant divergence of both sides yields:

$$\nabla^\mu T_{\mu\nu} = \nabla^\mu \left[\frac{1}{8\pi G} G_{\mu\nu} + \Lambda g_{\mu\nu} + \Delta^{\text{RG}}_{\mu\nu} \right]$$

6.2 Contribution from the Einstein Tensor

Using the Bianchi identity:

$$\nabla^\mu G_{\mu\nu} = 0$$

we obtain:

$$\nabla^\mu \left(\frac{1}{8\pi G} G_{\mu\nu} \right) = -\frac{1}{8\pi} \frac{\nabla^\mu G}{G^2} G_{\mu\nu}$$

6.3 Contribution from the Cosmological Term

We have:

$$\nabla^\mu(\Lambda g_{\mu\nu}) = \partial_\nu \Lambda$$

6.4 Contribution from RG Corrections

The tensor $\Delta^{\mathrm{RG}}_{\mu\nu}$ is constructed from gradients of G and from the variation of the RG scale $k[g]$.

Proposition 6.1. *Its divergence can be written as:*

$$\nabla^\mu \Delta^{\mathrm{RG}}_{\mu\nu} = \mathcal{F}_\nu(G, \nabla G, \nabla^2 G, \nabla k)$$

where $\mathcal{F}_\nu$ is a covariant vector depending on derivatives of the running couplings and on the variation of the RG scale.

Proof. The result follows from the explicit structure of $\Delta^{\mathrm{RG}}_{\mu\nu}$, which contains terms of the form $\nabla_\mu \nabla_\nu G^{-1}$ and contributions from $\Theta_{\mu\nu}$. Taking the divergence produces terms involving second and third derivatives of G and derivatives of $k[g]$. A detailed derivation is given in Appendix K. □

6.5 Modified Conservation Law

Proposition 6.2. *The covariant divergence of the energy-momentum tensor is:*

$$\nabla^\mu T_{\mu\nu} = -\frac{1}{8\pi}\frac{\nabla^\mu G}{G^2} G_{\mu\nu} + \partial_\nu \Lambda + \mathcal{F}_\nu$$

This shows that the matter energy-momentum tensor is not conserved independently when the couplings are spacetime-dependent.

6.6 Total Energy-Momentum Conservation

Proposition 6.3. *The total energy-momentum tensor, including RG contributions, is covariantly conserved:*

$$\nabla^\mu \left(T_{\mu\nu} - \frac{1}{8\pi G} G_{\mu\nu} - \Lambda g_{\mu\nu} - \Delta^{\mathrm{RG}}_{\mu\nu} \right) = 0$$

Proof. This follows from the variational origin of the field equations and general covariance of the action. The conservation law reflects the diffeomorphism invariance of the underlying theory. □

Remark. This conservation law is independent of the specific parametrization of the RG scale (Appendix O), and therefore represents a robust structural property of the theory.

6.7 Interpretation as Energy Exchange

The non-vanishing divergence of $T_{\mu\nu}$ indicates an exchange of energy-momentum between the matter sector and the RG-induced sector.

In particular:

- the matter energy-momentum tensor is not conserved in isolation,

- variations of G and Λ act as effective sources or sinks,
- the total system remains covariantly conserved.

6.8 Reduction to FLRW Cosmology

In a homogeneous and isotropic background, the conservation equation reduces to a modified continuity equation.

Proposition 6.4. *The energy density satisfies:*

$$\dot{\rho} + 3H(\rho + p) = \mathcal{S}_{\mathrm{RG}}$$

with:

$$\mathcal{S}_{\mathrm{RG}} = -\frac{\dot{G}}{G}\rho - \frac{\dot{\Lambda}}{8\pi G} + \Sigma_{\mathrm{RG}}$$

where Σ_{RG} encodes contributions from $\Theta_{\mu\nu}$.

Proof. The result follows by evaluating the time component of the conservation equation in the FLRW background and using the symmetry constraints of the metric. □

6.9 Consistency Limit

In the limit of constant couplings:

$$\dot{G} = 0, \quad \dot{\Lambda} = 0$$

all RG contributions vanish:

$$\mathcal{S}_{\mathrm{RG}} = 0$$

and the standard conservation law is recovered:

$$\dot{\rho} + 3H(\rho + p) = 0$$

6.10 Validity and Limitations

The modified conservation law relies on:

- the Einstein–Hilbert truncation of the effective action,
- the derivative expansion of $k[g]$ (Appendix P),
- the neglect of higher-order non-local effects.

Proposition 6.5. *Within this regime, the energy exchange terms provide a consistent effective description of the interaction between matter and the RG-induced sector.*

6.11 Physical Interpretation

The modified conservation law reflects the fact that the cosmological system is not closed at the level of matter fields.

In particular:

- variations of G induce an effective coupling between matter and geometry,

- variations of Λ correspond to a dynamical vacuum energy exchange,
- additional RG contributions encode higher-order and potentially non-local effects.

This structure provides the dynamical mechanism through which the renormalization group flow backreacts on cosmological evolution.

7 Cosmological Regimes

7.1 General Strategy

The modified cosmological equations derived in the previous sections define a dynamical system:

$$H(t), \quad \rho(t), \quad G(H), \quad \Lambda(H)$$

whose behavior is determined by the properties of the RG flow.

Rather than specifying explicit functional forms for $G(H)$ and $\Lambda(H)$, we analyze the asymptotic regimes that follow from general properties of the flow and its scaling behavior.

Remark. The dependence $G(H)$ and $\Lambda(H)$ is understood as arising from the composition $G(k[g])$, with $k \simeq k(H)$ at leading order (Section 2). This identification is valid within the derivative expansion regime (Appendix P) and is independent of the RG parametrization (Appendix O).

7.2 Ultraviolet Regime

At high energies, the RG flow is expected to approach a scaling regime. In functional renormalization group approaches to gravity [6, 7], one typically finds:

$$G(k) \sim \frac{1}{k^2}, \quad \Lambda(k) \sim k^2$$

up to scheme-dependent coefficients.

Using the dynamical identification $k \sim H$, this suggests:

$$G(H) \sim \frac{1}{H^2}, \quad \Lambda(H) \sim H^2$$

Proposition 7.1. *Under these scaling assumptions, the cosmological equations admit quasi-de Sitter solutions with approximately constant Hubble parameter.*

Proof. Substituting the scaling behavior into the modified Friedmann equation, the dominant contribution arises from $\Lambda(H) \sim H^2$. Neglecting subleading matter and derivative terms, one obtains a self-consistent solution with approximately constant H. $\square$

Validity. This analysis assumes that:

- the scaling regime extends to the relevant cosmological scales,
- higher-derivative corrections remain subleading,
- the derivative expansion remains under control (Appendix P).

7.3 Interpretation: RG-Driven Inflation

In this regime, the scale factor exhibits accelerated expansion:

$$a(t) \sim e^{Ht}$$

This provides a mechanism for inflation driven by the RG flow itself, without introducing an additional inflaton field. The accelerated expansion arises from the scaling behavior of the effective cosmological term.

Remark. The existence and duration of this phase depend on the detailed RG trajectory and should be understood as a dynamical regime rather than a universal prediction.

7.4 Transition Regime

As the universe expands, the Hubble parameter decreases and the system exits the ultraviolet scaling regime.

Proposition 7.2. *The transition occurs when matter contributions and RG-induced terms become comparable in the modified Friedmann equation.*

In this regime:

- the running of $G(H)$ slows down,
- the effective cosmological term $\Lambda(H)$ decreases,
- matter contributions become dynamically relevant.

This provides a dynamical mechanism for exiting the inflationary phase.

7.5 Intermediate Regime

At intermediate scales, the RG flow approaches a regime where variations of the couplings are suppressed.

Proposition 7.3. *If the RG flow satisfies:*

$$\frac{dG}{dH} \to 0, \quad \frac{d\Lambda}{dH} \to 0$$

the cosmological dynamics reduces to that of standard general relativity.

Proof. When derivatives of the couplings vanish, the RG-induced correction terms in the modified Friedmann and Raychaudhuri equations become negligible, recovering the standard equations. □

7.6 Interpretation

This regime corresponds to an effective classical phase in which:

- the gravitational coupling is approximately constant,
- the cosmological term is slowly varying,
- standard cosmological evolution is recovered.

7.7 Infrared Regime

At late times, the RG flow is expected to approach an infrared regime characterized by a slowly varying or asymptotically constant cosmological term.

Proposition 7.4. *If* $\Lambda(H) \to \Lambda_0 > 0$ *as* $H \to 0$*, the cosmological evolution admits a late-time accelerated expansion.*

Proof. In the limit of negligible matter density, the Friedmann equation reduces to:

$$H^2 \approx \frac{\Lambda_0}{3}$$

leading to exponential expansion. □

7.8 Interpretation: Emergent Dark Energy

In this regime, late-time acceleration arises as an infrared effect of the RG flow, with the effective cosmological term playing the role of dark energy.

Remark. The detailed behavior of $\Lambda(H)$ in the infrared depends on the RG trajectory and cannot be fixed without additional input.

7.9 Summary of Regimes

The cosmological dynamics exhibits a sequence of regimes:

- ultraviolet regime: RG-driven accelerated expansion,
- transition regime: exit from the scaling phase,
- intermediate regime: effective classical cosmology,
- infrared regime: late-time acceleration.

These regimes follow from general properties of the RG flow and its coupling to geometry, without specifying detailed functional forms for the running couplings.

8 Flow Saturation and Dynamical Instabilities

8.1 Motivation

The renormalization group (RG) flow governs the scale dependence of the effective couplings $G(k)$ and $\Lambda(k)$. In a cosmological setting, the RG scale is dynamically determined by the geometry (Section 2), leading to couplings of the form:

$$G = G(H), \quad \Lambda = \Lambda(H)$$

While the ultraviolet (UV) and infrared (IR) regimes are characterized by asymptotic scaling behaviors, the intermediate regime is expected to exhibit non-trivial structure.

Proposition 8.1. *Under general assumptions on the spectral density of fluctuations, the RG flow cannot exhibit unbounded growth and must admit regimes in which the running of couplings is effectively suppressed.*

Sketch. The RG flow is controlled by the spectral density of the fluctuation operator (Appendix Q). Since the number of effective modes contributing to the flow is finite at each scale and grows at most polynomially, the flow cannot sustain arbitrarily large variations indefinitely. This leads generically to regimes where the beta functions decrease in magnitude. □

This motivates the introduction of a saturation regime and the analysis of its dynamical consequences.

8.2 Definition of Flow Saturation

Definition 8.2. *We say that the RG flow saturates in a regime if:*

$$\frac{dG}{dk} \to 0, \quad \frac{d\Lambda}{dk} \to 0 \quad \textit{as } k \to k_*$$

for some characteristic scale k_*.

Using the dynamical identification $k = k(H)$ (valid within the derivative expansion, Appendix P), this implies:

$$\frac{dG}{dH} \to 0, \quad \frac{d\Lambda}{dH} \to 0$$

Remark. This condition does not imply that the couplings are strictly constant, but rather that their variation becomes parametrically suppressed.

8.3 Effective Dynamical System

The cosmological evolution is governed by:

$$H^2 = \frac{8\pi}{3}G(H)\rho + \frac{\Lambda(H)}{3} + \Delta_{RG}$$
$$\dot{H} = -4\pi G(H)(\rho + p) + \Xi_{RG}$$

where Δ_{RG} and Ξ_{RG} encode corrections arising from the running couplings (Section 5).

We consider perturbations around a background solution:

$$H(t) = H_0(t) + \delta H(t), \quad \delta H \ll H_0$$

in a regime where the derivative expansion remains valid (Appendix P).

8.4 Expansion of Running Couplings

We expand the couplings around H_0:

$$G(H) = G_0 + G_0'\delta H + \frac{1}{2}G_0''(\delta H)^2 + \cdots$$
$$\Lambda(H) = \Lambda_0 + \Lambda_0'\delta H + \frac{1}{2}\Lambda_0''(\delta H)^2 + \cdots$$

Their time derivatives satisfy:

$$\dot{G} = G_0'\dot{H} + \mathcal{O}(\delta H)$$
$$\delta\dot{G} = G_0''\dot{H}_0\,\delta H + G_0'\delta\dot{H}$$

8.5 Linearization of the Dynamical Equations

We linearize the Raychaudhuri equation:

$$\dot{H} = -4\pi G(H)(\rho + p) + \Xi_{RG}$$

At first order:

$$\delta\dot{H} = -4\pi(\rho + p)\delta G + \delta\Xi_{RG}.$$

Using:

$$\delta G = G_0'\delta H$$

we obtain:

$$\delta\dot{H} = -4\pi G_0'(\rho + p)\,\delta H + \delta\Xi_{RG}$$

8.6 Structure of RG Contributions

The RG correction Ξ_{RG} depends on derivatives of $G(H)$:

$$\Xi_{RG} \sim \ddot{G}^{-1} + H\dot{G}^{-1} + \cdots$$

Expanding to linear order yields:

$$\delta\Xi_{RG} = \mathcal{A}_1(t)\,\delta H + \mathcal{A}_2(t)\,\delta\dot{H}$$

where $\mathcal{A}_1$ and $\mathcal{A}_2$ depend on G_0', G_0'', $\dot{H}_0$, and higher derivatives.
Substituting:

$$\delta\dot{H} = -4\pi G_0'(\rho + p)\,\delta H + \mathcal{A}_1\delta H + \mathcal{A}_2\delta\dot{H}$$

8.7 Effective Linear Equation

Rearranging:

$$(1 - \mathcal{A}_2)\delta\dot{H} = \left[-4\pi G_0'(\rho + p) + \mathcal{A}_1\right]\delta H$$

Assuming $1 - \mathcal{A}_2 \neq 0$, we obtain:

$$\delta\dot{H} = \mathcal{A}(t)\,\delta H$$

with:

$$\mathcal{A}(t) = \frac{-4\pi G_0'(\rho + p) + \mathcal{A}_1(t)}{1 - \mathcal{A}_2(t)}$$

8.8 Instability Criterion

Theorem 8.3 (Dynamical Instability)**.** *If $\mathcal{A}(t) > 0$ over a finite time interval, perturbations grow exponentially:*

$$\delta H(t) \sim \exp\left(\int \mathcal{A}(t)\,dt\right)$$

and the background solution is dynamically unstable.

Proof. The result follows directly from the linear differential equation for δH. □

8.9 Relation to Flow Curvature

Proposition 8.4. *In regimes where matter contributions are subdominant, a sufficient condition for instability is:*

$$G''(H) > 0 \quad or \quad \Lambda''(H) > 0$$

Proof. The coefficients $\mathcal{A}_1$ and $\mathcal{A}_2$ receive contributions proportional to second derivatives of the couplings. Positive curvature enhances the effective growth rate $\mathcal{A}(t)$ and can dominate over damping terms. □

8.10 Physical Interpretation

In the saturation regime:

$$G'(H) \to 0, \quad \Lambda'(H) \to 0$$

while higher derivatives remain generically non-zero.

As a result:

- the background evolution approaches an effective quasi-fixed regime,
- perturbations probe the curvature of the RG flow,
- convex regions of the flow lead to amplification of fluctuations.

Thus, saturation naturally induces dynamical instabilities.

8.11 Towards Structure Formation

The instability mechanism implies that:

- homogeneous cosmological solutions can become unstable,
- fluctuations may grow without introducing additional dynamical fields,
- the onset of structure formation can be triggered by RG-induced dynamics.

Remark. A full nonlinear treatment is required to describe structure formation quantitatively. The present analysis establishes the existence of an instability mechanism at the linear level.

8.12 Discussion

The mechanism identified here differs from standard cosmological scenarios:

- it does not rely on a separate inflaton sector,
- it is driven by intrinsic properties of the RG flow,
- it connects microscopic spectral properties to macroscopic structure formation.

This provides a novel perspective on the origin of cosmological inhomogeneities within the present framework.

9 Quantitative Implications

9.1 RG-Induced Correction to Cosmological Dynamics

The modified field equations derived in Section 4 provide a closed description of gravitational dynamics in the presence of scale-dependent couplings.

In a homogeneous and isotropic background, the RG-induced tensor $\Theta_{\mu\nu}$ can be written as an effective perfect fluid:

$$\Theta_{\mu\nu} = (\rho_{RG} + p_{RG})u_\mu u_\nu + p_{RG} g_{\mu\nu}$$

where $\rho_{RG}(H)$ and $p_{RG}(H)$ are determined by the running of the couplings and the functional dependence $k[g]$.

Substituting into the (00) component of the field equations yields:

$$H^2 = \frac{8\pi G(H)}{3}\rho + \frac{\Lambda(H)}{3} + \Delta_{RG}(H, \dot{H})$$

where Δ_{RG} encodes RG-induced corrections arising from both the variation of $G(H)$ and the tensor $\Theta_{\mu\nu}$.

9.2 Origin and Structure of the RG Correction

The dominant contribution to Δ_{RG} originates from the variation of the RG scale in the cosmological term. Using the results of Section 4, one obtains schematically:

$$\Delta_{RG} \sim \frac{1}{3}\frac{d\Lambda}{dk}\frac{dk}{dH}\mathcal{D}[H]$$

where $\mathcal{D}[H]$ arises from the variation of the Hubble parameter.

In a FLRW background, the structure of δH leads to:

$$\mathcal{D}[H] \sim H$$

so that:

$$\Delta_{RG} \sim \frac{1}{3}\frac{d\Lambda}{dk}\frac{dk}{dH}H$$

Origin of the H factors. The scaling of Δ_{RG} results from three independent contributions:

- one power of H from the variation of the expansion rate (δH),
- one power from the RG running through $d\Lambda/dk$,
- one power from the spectral identification $k \sim H$ (Section 2).

This leads to a cubic scaling under suitable conditions.

9.3 Scaling Regime and Emergence of H^3

Assuming a scaling regime of the form:

$$\Lambda(k) = \lambda_* k^2$$

one has:

$$\frac{d\Lambda}{dk} = 2\lambda_* k$$

Using the dynamical identification $k \sim H$, and $\frac{dk}{dH} \sim 1$, one obtains:

$$\Delta_{RG} \sim \lambda_* H^3$$

Proposition 9.1. *In regimes where $\Lambda(k) \propto k^2$ and the derivative expansion is valid, the leading RG-induced correction scales as:*

$$\Delta_{RG} \sim H^3$$

Remark (Non-universality). The H^3 scaling is not universal. It relies on:

- the approximate relation $k \sim H$,
- a quadratic scaling of $\Lambda(k)$,
- the truncation of higher-derivative contributions.

Different RG trajectories may lead to different functional forms.

9.4 Magnitude of the Correction

The correction can be written as:

$$\Delta_{RG} = \gamma(H) H^3$$

with:

$$\gamma(H) \sim \frac{1}{H} \frac{d\Lambda}{dk}$$

In a scaling regime $\Lambda(k) = \lambda_* k^2$, one obtains:

$$\gamma(H) \simeq 2\lambda_*$$

Estimate. Functional RG studies typically yield:

$$\lambda_* = \mathcal{O}(0.1 - 1)$$

so that:

$$\gamma(H) = \mathcal{O}(0.1 - 1)$$

Thus, the correction is naturally unsuppressed and does not require fine-tuning.

9.5 Modified Friedmann Equation

The effective cosmological equation becomes:

$$\boxed{H^2 = \frac{8\pi G(H)}{3}\rho + \frac{\Lambda(H)}{3} + \gamma(H)\, H^3}$$

This defines an implicit cubic equation:

$$\gamma(H) H^3 - H^2 + H_{\text{std}}^2 = 0$$

where H_{std} denotes the standard ΛCDM contribution.

Branch selection. Multiple solutions may exist. The physically relevant branch is defined by continuity with the standard cosmological solution at low redshift.

9.6 Observable Consequences

Defining:

$$\tilde{\gamma}(H) = \gamma(H)H$$

the modified equation can be written as:

$$H^2(1 - \tilde{\gamma}) = H_{\rm std}^2$$

Deviations from ΛCDM become significant when:

$$\gamma(H)\,H \sim 1$$

This condition defines the regime in which RG effects dominate the expansion.

9.7 Impact on the Expansion History

The modified equation leads to a deformation of the expansion rate $H(z)$:

- at low redshift ($H \ll 1/\gamma$), standard ΛCDM behavior is recovered,
- at intermediate redshift, corrections remain perturbative,
- at high redshift ($H \gtrsim 1/\gamma$), the expansion rate is enhanced.

Thus:

$$H(z) > H_{\rm std}(z) \quad \text{for sufficiently large } z$$

9.8 Implications for High-Redshift Observations

An enhanced expansion rate modifies the relation between cosmic time and redshift:

$$dt \sim \frac{dz}{(1+z)H(z)}$$

As a consequence:

- cosmic time at fixed redshift is reduced,
- effective energy densities are modified,
- the growth history of structures is altered.

This may affect the interpretation of observations of early massive galaxies and high-redshift structures.

Caveat. A definitive assessment requires a perturbative analysis beyond the homogeneous background.

9.9 Relation to Cosmological Regimes

The RG-induced correction fits consistently within the global picture:

- in the ultraviolet regime, the H^3 term enhances accelerated expansion,
- in the intermediate regime, it becomes subleading,
- in the infrared regime, it is negligible compared to $\Lambda(H)$.

This ensures continuity with standard cosmology.

9.10 Limitations

The present analysis relies on:

- the Einstein–Hilbert truncation,
- the identification $k \sim H$,
- the derivative expansion of $k[g]$ (Appendix P),
- the restriction to homogeneous cosmology.

In particular, the impact of RG corrections on perturbations and structure formation remains to be determined.

9.11 Outlook

The framework suggests several directions:

- numerical integration of the modified Friedmann equation,
- inclusion of perturbations and growth of structures,
- confrontation with cosmological observations,
- extension beyond FLRW symmetry.

The presence of a H^3 correction provides a concrete and testable signature of RG-induced cosmological dynamics.

10 Discussion and Physical Implications

10.1 Inflation without an External Scalar Sector

In the ultraviolet regime, the scaling properties of the RG flow can generate an effective cosmological term of order H^2.

Proposition 10.1. *Under these conditions, the cosmological equations admit accelerated expansion phases without introducing an additional scalar field.*

This implies that inflation can arise as a dynamical regime of the RG flow, with its duration and exit controlled by the evolution toward lower-energy regimes.

Remark. This mechanism depends on the existence of an appropriate scaling regime and should be understood as a dynamical possibility rather than a universal prediction.

10.2 Structure Formation from RG-Induced Instabilities

The saturation regime identified in Section 7 provides a mechanism for dynamical instabilities:

- first derivatives of the couplings are suppressed,
- second derivatives remain non-zero,
- perturbations probe the curvature of the RG flow,
- convex regions lead to amplification.

Proposition 10.2. *Under these conditions, homogeneous cosmological solutions can become dynamically unstable.*

This suggests that the emergence of inhomogeneities may be linked to intrinsic properties of the RG flow.

Remark. The present analysis is limited to linear perturbations; a nonlinear treatment is required for quantitative predictions.

10.3 Dark Energy as an Infrared Effect

In the infrared regime, the RG flow can approach configurations where $\Lambda(H)$ tends toward a small positive value.

Proposition 10.3. *In such regimes, late-time acceleration arises as an emergent effect of the RG flow.*

This provides a dynamical interpretation of dark energy, with its value determined by the RG trajectory rather than fixed at the level of the action.

Remark. The detailed behavior depends on the full RG flow and cannot be fixed within the present truncation.

11 Conclusion

In this work, we have established a direct connection between renormalization group (RG) flow and cosmological dynamics within the framework of Information Flow Theory.

Starting from a scalar-induced framework in which geometry and effective interactions emerge dynamically, we have shown that the RG scale can be related to the spectral properties of the spacetime geometry. Within a controlled derivative expansion (Appendix P), this leads to running gravitational couplings $G(H)$ and $\Lambda(H)$, and to modified field equations incorporating RG-induced corrections.

From these equations, we have derived a consistent set of cosmological dynamics, including modified Friedmann and Raychaudhuri equations, together with a generalized conservation law reflecting energy exchange between matter and the RG-induced sector.

A central result of this work is that qualitatively distinct cosmological regimes emerge from general properties of the RG flow. In particular:

- accelerated expansion can arise in the ultraviolet regime under appropriate scaling behavior of the flow, without the introduction of an additional scalar sector,
- a classical cosmological phase is recovered when the running of the couplings becomes negligible,
- late-time acceleration can emerge as an infrared effect associated with the RG trajectory.

In addition, we have identified a dynamical instability mechanism associated with regimes of flow saturation. In such regimes, the suppression of first derivatives of the running couplings, combined with non-vanishing higher derivatives, leads to an amplification of perturbations. This provides a mechanism through which cosmological inhomogeneities may originate from intrinsic properties of the RG flow.

The present analysis is based on controlled approximations. In particular:

- the effective action is treated at the level of the Einstein–Hilbert truncation,
- higher-order curvature terms and non-local contributions are neglected,
- the identification $k = k[g]$ is derived within a derivative expansion and is not obtained from a fully non-perturbative construction,
- the nonlinear evolution of RG-induced instabilities is not addressed.

These limitations restrict the quantitative predictive power of the framework but do not affect the qualitative mechanisms identified in this work.

Overall, the results presented here define a coherent and mathematically controlled setting in which cosmological evolution is directly linked to renormalization group properties. They suggest that phenomena such as inflation, late-time acceleration, and the emergence of inhomogeneities may admit a unified interpretation in terms of RG flow dynamics.

Further work will be required to extend the analysis beyond the present truncation, to develop quantitative predictions, and to confront the framework with observational data.

A Spectral Analysis in FLRW Background

A.1 Elliptic Operators and Spectral Scale

We consider a second-order differential operator of the form:

$$\mathcal{O} = -\nabla^\mu \nabla_\mu + \mathcal{U}(x)$$

acting on scalar fluctuations.

The spectrum of $\mathcal{O}$ determines the dominant contributions to the functional trace appearing in the RG flow:

$$\mathrm{Tr}\left[(\Gamma_k^{(2)} + R_k)^{-1}\right]$$

Definition A.1. *We define the effective spectral scale λ_{eff} as the characteristic magnitude of eigenvalues contributing dominantly to the trace.*

A.2 FLRW Geometry

We consider a spatially flat FLRW spacetime:

$$ds^2 = -dt^2 + a(t)^2 d\vec{x}^2$$

The covariant Laplacian acting on a scalar field ϕ is:

$$\nabla^\mu \nabla_\mu \phi = -\ddot{\phi} - 3H\dot{\phi} + \frac{1}{a^2}\Delta\phi$$

where $H = \dot{a}/a$.

A.3 Mode Decomposition

We expand the field in Fourier modes:

$$\phi(t, \vec{x}) = \int d^3k \, \phi_k(t) e^{i\vec{k}\cdot\vec{x}}$$

Each mode satisfies:

$$\ddot{\phi}_k + 3H\dot{\phi}_k + \frac{k^2}{a^2}\phi_k + \mathcal{U}(t)\phi_k = 0$$

A.4 Effective Frequency

We rewrite the equation as:

$$\ddot{\phi}_k + \omega_k^2(t)\phi_k = 0$$

with effective frequency:

$$\omega_k^2(t) = \frac{k^2}{a^2} + \mathcal{U}(t) - \frac{3}{2}\dot{H} - \frac{9}{4}H^2$$

Proposition A.2. *The effective spectral scale is determined by:*

$$\lambda_{\mathrm{eff}}(k, t) \sim \omega_k^2(t)$$

A.5 Curvature Contributions

The curvature scalar in FLRW is:

$$R = 6(2H^2 + \dot{H})$$

In general, the effective potential $\mathcal{U}(t)$ contains curvature contributions:

$$\mathcal{U}(t) = \alpha R + \cdots$$

with α a constant depending on the model.

Thus:

$$\mathcal{U}(t) \sim H^2 + \dot{H}$$

A.6 Dominant Spectral Regime

We distinguish two regimes:

- UV modes: $\frac{k^2}{a^2} \gg H^2$
- IR modes: $\frac{k^2}{a^2} \lesssim H^2$

The RG flow at scale k is dominated by modes satisfying:

$$\frac{k^2}{a^2} \sim \lambda_{\text{eff}}$$

A.7 Identification of the Spectral Scale

In the infrared-dominated regime relevant for cosmological evolution, spatial momenta are redshifted:

$$\frac{k^2}{a^2} \ll H^2$$

Thus, the dominant contribution to ω_k^2 comes from curvature terms:

$$\omega_k^2 \sim H^2 + |\dot{H}|$$

Theorem A.3. *The effective spectral scale in FLRW spacetime satisfies:*

$$\lambda_{\text{eff}} \sim H^2 + |\dot{H}|$$

Proof. In the IR regime, the dominant contributions to the operator $\mathcal{O}$ arise from curvature-dependent terms and time derivatives of the metric. Since these scale as H^2 and $\dot{H}$, they set the characteristic eigenvalue scale. □

A.8 Implication for the RG Scale

Matching the RG scale to the dominant spectral scale yields:

$$k^2 \sim \lambda_{\text{eff}} \sim H^2 + |\dot{H}|$$

This relation provides the dynamical identification of the RG scale used throughout the main text.

B RG-Induced Effective Action

B.1 Functional Renormalization Group Equation

The effective average action $\Gamma_k[g]$ satisfies the Wetterich equation:

$$\partial_k \Gamma_k = \frac{1}{2} \mathrm{Tr} \left[(\Gamma_k^{(2)} + R_k)^{-1} \partial_k R_k \right]$$

This equation describes the scale dependence of the effective action under successive integration of modes.

B.2 Local Expansion of the Effective Action

Under general assumptions of locality and covariance, the effective action admits an expansion in local geometric invariants:

$$\Gamma_k[g] = \int d^4x \sqrt{-g} \left[\mathcal{L}_k(g, R, \nabla R, R^2, \dots) \right]$$

Proposition B.1. *The Lagrangian density can be organized as a derivative expansion:*

$$\mathcal{L}_k = U_k + Z_k R + \sum_i c_i(k) \mathcal{O}_i(R^2, \nabla R, \dots)$$

where $\mathcal{O}_i$ are higher-order curvature invariants.

Proof. The result follows from general covariance: all scalar terms must be constructed from curvature invariants and their derivatives. Locality ensures that only a finite number of derivatives contribute at a given order. □

B.3 Canonical Dimensions and Scaling

Each term in the expansion has a definite mass dimension:

- U_k has dimension k^4,
- R has dimension k^2,
- higher-order terms involve additional powers of k^{-2}.

Proposition B.2. *At scales $k \ll \Lambda_{\mathrm{UV}}$, higher-order curvature terms are suppressed relative to R.*

Proof. Dimensional analysis implies that terms such as R^2 enter with coefficients proportional to inverse powers of k^2. In the infrared regime, these contributions are suppressed compared to the linear curvature term. □

B.4 Infrared Truncation

Theorem B.3. *In the infrared regime, the effective action reduces to:*

$$\Gamma_k[g] = \int d^4x \sqrt{-g} \left[U_k + Z_k R \right] + \mathcal{O}(R^2)$$

Proof. The suppression of higher-order terms established above implies that the leading contributions are given by the constant and linear curvature terms. □

B.5 Identification of Gravitational Couplings

We define:

$$\frac{1}{16\pi G(k)} = Z_k, \quad \Lambda(k) = -U_k$$

Proposition B.4. *The effective action takes the Einstein–Hilbert form:*

$$\Gamma_k[g] = \int d^4x\sqrt{-g}\left[\frac{1}{16\pi G(k)}R - \Lambda(k)\right]$$

B.6 Consistency with RG Flow

The running of $G(k)$ and $\Lambda(k)$ is determined by the Wetterich equation projected onto the corresponding operators.

Proposition B.5. *The RG flow preserves the structure of the truncated action under projection onto the* $(1, R)$ *subspace.*

Proof. Projecting the flow equation onto the basis of invariants $\{1, R\}$ yields flow equations for U_k and Z_k. This defines a closed system at the level of the truncation. □

B.7 Domain of Validity

The Einstein–Hilbert truncation is valid under the following conditions:

- curvature invariants satisfy $R \ll k^2$,
- derivatives of curvature are small compared to k,
- the flow remains within a regime where higher operators are suppressed.

Proposition B.6. *In slowly varying cosmological backgrounds, these conditions are satisfied in the intermediate and infrared regimes.*

B.8 Interpretation

The RG flow dynamically generates an effective gravitational action whose leading structure coincides with the Einstein–Hilbert action.

The gravitational coupling and cosmological constant are not fundamental parameters but scale-dependent quantities arising from the underlying scalar dynamics.

This provides a non-perturbative origin for gravitational interactions within the present framework.

C Functional Dependence of the RG Scale

C.1 Spectral Origin of the RG Scale

The renormalization group flow is governed by the functional trace:

$$\partial_k\Gamma_k = \frac{1}{2}\mathrm{Tr}\left[(\Gamma_k^{(2)} + R_k)^{-1}\partial_k R_k\right]$$

The dominant contributions to the trace arise from eigenvalues of $\Gamma_k^{(2)}$ satisfying:

$$\lambda_n \sim k^2$$

Definition C.1. *We define the effective spectral scale λ_{eff} as the characteristic value of eigenvalues contributing significantly to the trace.*

C.2 Dependence on the Background Geometry

The operator $\Gamma_k^{(2)}$ is constructed from covariant derivatives and curvature tensors. Therefore, its spectrum depends on the background metric:

$$\lambda_n = \lambda_n[g_{\mu\nu}]$$

Proposition C.2. *The effective spectral scale is a functional of the metric:*

$$\lambda_{\text{eff}} = \lambda_{\text{eff}}[g]$$

Proof. Since the eigenvalue problem for $\Gamma_k^{(2)}$ depends explicitly on the metric through covariant derivatives and curvature, the spectrum varies under changes of $g_{\mu\nu}$. The dominant eigenvalues therefore define a functional of the geometry. □

C.3 Implicit Definition of the RG Scale

The RG flow is dominated by modes satisfying:

$$\lambda_{\text{eff}}[g] \sim k^2$$

Definition C.3. *We define the RG scale $k[g]$ implicitly by:*

$$k[g]^2 = \lambda_{\text{eff}}[g]$$

Proposition C.4. *This defines the RG scale as a functional of the metric.*

C.4 Covariance

Proposition C.5. *The functional $k[g]$ is diffeomorphism-invariant.*

Proof. The operator $\Gamma_k^{(2)}$ is constructed covariantly, and its spectrum depends only on geometric invariants. Therefore, $\lambda_{\text{eff}}[g]$ is invariant under coordinate transformations, and so is $k[g]$. □

C.5 Local Approximation

In general, $k[g]$ is a non-local functional. However, in slowly varying backgrounds, it admits a local approximation.

Proposition C.6. *In the derivative expansion, the spectral scale can be approximated by local curvature invariants:*

$$k^2(x) \approx c_1 R(x) + c_2 \frac{\nabla^2 R}{R} + \cdots$$

Proof. The heat kernel expansion of elliptic operators expresses the trace in terms of local curvature invariants. Truncating this expansion yields a local approximation for the spectral scale. □

C.6 Application to FLRW Geometry

In FLRW spacetime, curvature invariants reduce to:

$$R \sim H^2 + \dot{H}$$

Proposition C.7. *In a homogeneous and isotropic background, the RG scale satisfies:*

$$k^2 \sim H^2 + |\dot{H}|$$

Proof. Substituting the FLRW expressions for curvature invariants into the local approximation yields the stated relation. □

C.7 Consistency with the RG Flow

Proposition C.8. *The identification $k = k[g]$ is consistent with the structure of the RG flow.*

Proof. The RG equation is dominated by modes with eigenvalues $\lambda_n \sim k^2$. Identifying k with the spectral scale ensures that the flow parameter matches the dynamically relevant modes. □

C.8 Interpretation

The RG scale is not an external parameter but an intrinsic property of the system, determined by the spectral structure of the effective operator.

This establishes a dynamical link between geometry and renormalization flow, forming the basis of the cosmological construction developed in the main text.

D Variation of the Einstein–Hilbert Action with Running Couplings

D.1 Action with Position-Dependent Coupling

We consider the action:

$$S[g] = \int d^4x \sqrt{-g}\, f(x) R,$$

where:

$$f(x) = \frac{1}{16\pi G(x)}$$

is a scalar function depending on spacetime through the RG flow.

D.2 Variation of the Determinant

The variation of the metric determinant is:

$$\delta\sqrt{-g} = -\frac{1}{2}\sqrt{-g}\, g_{\mu\nu}\, \delta g^{\mu\nu}$$

D.3 Variation of the Ricci Scalar

The variation of the Ricci scalar is:

$$\delta R = R_{\mu\nu}\, \delta g^{\mu\nu} + g^{\mu\nu} \delta R_{\mu\nu}$$

Using the identity:

$$\delta R_{\mu\nu} = \nabla_\lambda \delta\Gamma^\lambda_{\mu\nu} - \nabla_\nu \delta\Gamma^\lambda_{\mu\lambda}$$

we obtain:

$$g^{\mu\nu}\delta R_{\mu\nu} = \nabla_\lambda V^\lambda$$

where:

$$V^\lambda = g^{\mu\nu}\delta\Gamma^\lambda_{\mu\nu} - g^{\lambda\nu}\delta\Gamma^\mu_{\mu\nu}$$

D.4 Variation of the Action

We compute:

$$\delta S = \int d^4x \left[\delta(\sqrt{-g})fR + \sqrt{-g}f\delta R + \sqrt{-g}R\delta f\right]$$

Substituting:

$$\delta S = \int d^4x\sqrt{-g}\left[-\frac{1}{2}g_{\mu\nu}fR\,\delta g^{\mu\nu} + fR_{\mu\nu}\delta g^{\mu\nu} + f\nabla_\lambda V^\lambda + R\delta f\right]$$

D.5 Integration by Parts

We rewrite the divergence term:

$$\int \sqrt{-g}f\nabla_\lambda V^\lambda = \int \sqrt{-g}\nabla_\lambda(fV^\lambda) - \int \sqrt{-g}(\nabla_\lambda f)V^\lambda$$

The first term is a boundary term and is discarded.

Thus:

$$\delta S = \int d^4x\sqrt{-g}\left[fR_{\mu\nu} - \frac{1}{2}fRg_{\mu\nu}\right]\delta g^{\mu\nu} - \int d^4x\sqrt{-g}(\nabla_\lambda f)V^\lambda + \int d^4x\sqrt{-g}R\delta f$$

D.6 Evaluation of the Remaining Term

Using the explicit form of V^λ, one obtains:

$$-(\nabla_\lambda f)V^\lambda = (\nabla_\mu\nabla_\nu f - g_{\mu\nu}\Box f)\delta g^{\mu\nu}$$

D.7 Final Result

Combining all terms, we obtain:

Theorem D.1. *The variation of the action is:*

$$\delta S = \int d^4x\sqrt{-g}\left[fG_{\mu\nu} + \nabla_\mu\nabla_\nu f - g_{\mu\nu}\Box f\right]\delta g^{\mu\nu} + \int d^4x\sqrt{-g}R\delta f$$

D.8 Interpretation

The result differs from the standard Einstein–Hilbert variation by the presence of:

- derivative terms involving $f(x)$,

- an additional contribution proportional to $R\delta f$.

The latter term reflects the fact that f depends on the metric through the RG scale, and must be treated separately in the full variation of the effective action.

E Variation of the Cosmological Term

E.1 Action with Running Cosmological Term

We consider the contribution:

$$S_\Lambda[g] = -\int d^4x\sqrt{-g}\,\Lambda(k[g])$$

where Λ depends on the RG scale, itself a functional of the metric.

E.2 Variation of the Action

We compute:

$$\delta S_\Lambda = -\int d^4x\,\left[\delta\sqrt{-g}\,\Lambda + \sqrt{-g}\,\delta\Lambda\right]$$

E.3 Variation of the Determinant

Using:

$$\delta\sqrt{-g} = -\frac{1}{2}\sqrt{-g}g_{\mu\nu}\delta g^{\mu\nu}$$

we obtain:

$$-\delta\sqrt{-g}\,\Lambda = \frac{1}{2}\sqrt{-g}\Lambda g_{\mu\nu}\delta g^{\mu\nu}$$

E.4 Variation of the Running Coupling

Since $\Lambda = \Lambda(k[g])$, we apply the chain rule:

$$\delta\Lambda = \frac{d\Lambda}{dk}\,\delta k$$

Thus:

$$-\sqrt{-g}\delta\Lambda = -\sqrt{-g}\frac{d\Lambda}{dk}\delta k$$

E.5 Functional Variation of the RG Scale

The variation of the RG scale is given by:

$$\delta k = \int d^4x'\,\frac{\delta k}{\delta g_{\alpha\beta}(x')}\,\delta g_{\alpha\beta}(x')$$

Substituting into the action variation yields:

$$-\int d^4x\sqrt{-g}\frac{d\Lambda}{dk}\delta k = -\int d^4x\int d^4x'\,\sqrt{-g(x)}\frac{d\Lambda}{dk}(x)\frac{\delta k(x)}{\delta g_{\alpha\beta}(x')}\delta g_{\alpha\beta}(x')$$

E.6 Rewriting as a Local Contribution

We rewrite the variation in the form:

$$\delta S_\Lambda = \int d^4x\sqrt{-g}\left[\frac{1}{2}\Lambda g_{\mu\nu} + \Pi_{\mu\nu}\right]\delta g^{\mu\nu}$$

where:

$$\Pi_{\mu\nu}(x) = -\int d^4x'\,\sqrt{-g(x')}^{-1}\sqrt{-g(x)}\frac{d\Lambda}{dk}(x')\frac{\delta k(x')}{\delta g^{\mu\nu}(x)}$$

E.7 Structure of the Additional Term

Proposition E.1. *The tensor $\Pi_{\mu\nu}$ encodes the dependence of the cosmological term on the RG scale and is non-local in general.*

Proof. The variation involves a functional derivative $\delta k/\delta g_{\mu\nu}$, which depends on the global spectral properties of the geometry, leading to a non-local expression. □

E.8 Local Approximation

In slowly varying backgrounds, the RG scale admits a local approximation:

$$k = k(g, R, \nabla R, \dots)$$

Proposition E.2. *Under this approximation, $\Pi_{\mu\nu}$ reduces to a local tensor constructed from curvature invariants and derivatives of k.*

E.9 Final Result

Theorem E.3. *The variation of the cosmological term is:*

$$\delta S_\Lambda = \int d^4x\sqrt{-g}\left[\frac{1}{2}\Lambda g_{\mu\nu} + \Pi_{\mu\nu}\right]\delta g^{\mu\nu}$$

where $\Pi_{\mu\nu}$ encodes the implicit dependence of Λ on the metric through the RG scale.

E.10 Interpretation

The first term reproduces the standard cosmological contribution to the field equations.

The second term represents an additional contribution arising from the running of the cosmological constant, and must be included in the RG correction tensor of the main text.

F Variation of the RG Scale

F.1 Implicit Definition of the RG Scale

The RG scale is defined implicitly by:

$$k[g]^2 = \lambda_{\text{eff}}[g]$$

where λ_{eff} denotes the characteristic spectral scale of the operator $\Gamma_k^{(2)}$.

F.2 Variation of the RG Scale

Taking the variation, we obtain:

$$2k\,\delta k = \delta\lambda_{\text{eff}}$$

Thus:

$$\delta k = \frac{1}{2k}\,\delta\lambda_{\text{eff}}$$

F.3 Spectral Variation

Let $\mathcal{O}[g] = \Gamma_k^{(2)}$ be the relevant operator, with eigenvalues λ_n and eigenfunctions ψ_n:

$$\mathcal{O}\psi_n = \lambda_n\psi_n$$

Proposition F.1. *The variation of an eigenvalue is given by:*

$$\delta\lambda_n = \langle\psi_n|\delta\mathcal{O}|\psi_n\rangle$$

assuming normalized eigenfunctions.

Proof. This follows from standard perturbation theory for self-adjoint operators. □

F.4 Effective Spectral Variation

The effective scale λ_{eff} is determined by a weighted contribution of eigenvalues near the RG scale.

Proposition F.2. *The variation of the effective spectral scale can be expressed as:*

$$\delta\lambda_{\text{eff}} = \sum_n w_n\,\delta\lambda_n$$

where w_n are weight factors localized around $\lambda_n \sim k^2$.

Thus:

$$\delta\lambda_{\text{eff}} = \sum_n w_n\langle\psi_n|\delta\mathcal{O}|\psi_n\rangle$$

F.5 Variation of the Operator

The operator $\mathcal{O}$ depends on the metric through covariant derivatives and curvature terms:

$$\mathcal{O} = -\nabla^2 + \mathcal{U}(g)$$

Its variation takes the form:

$$\delta\mathcal{O} = \delta(-\nabla^2) + \delta\mathcal{U}$$

F.6 Variation of the Laplacian

The variation of the Laplacian acting on a scalar field is:

$$\delta(\nabla^2\phi) = -\delta g^{\mu\nu}\nabla_\mu\nabla_\nu\phi + \text{terms involving derivatives of } \delta g$$

Thus:

$$\delta(-\nabla^2) = \delta g^{\mu\nu}\nabla_\mu\nabla_\nu + \cdots$$

F.7 Structure of the Variation

Combining the above, we obtain:

$$\delta\lambda_{\text{eff}} = \int d^4x \sqrt{-g}\, \mathcal{K}^{\mu\nu}(x)\, \delta g_{\mu\nu}(x)$$

where $\mathcal{K}^{\mu\nu}$ is a tensor encoding spectral contributions.

Thus:

$$\delta k = \frac{1}{2k} \int d^4x \sqrt{-g}\, \mathcal{K}^{\mu\nu}(x)\, \delta g_{\mu\nu}(x)$$

F.8 Functional Derivative

We identify:

$$\frac{\delta k}{\delta g_{\mu\nu}(x)} = \frac{1}{2k} \sqrt{-g}\, \mathcal{K}^{\mu\nu}(x)$$

F.9 Local Approximation

In slowly varying backgrounds, the spectral tensor admits a local expansion:

$$\mathcal{K}^{\mu\nu} = a_1 g^{\mu\nu} H^2 + a_2 g^{\mu\nu} \dot{H} + \cdots .$$

Proposition F.3. *Under this approximation:*

$$\delta k \sim \frac{1}{k}(H\delta H + \delta\dot{H})$$

F.10 Interpretation

The RG scale responds to variations of the metric through the spectral properties of the effective operator.

The variation $\delta k[g]$ is:

- non-local at the fundamental level,
- expressible as a functional derivative involving spectral data,
- approximated locally in slowly varying cosmological backgrounds.

This structure justifies the presence of higher-derivative contributions in the field equations derived in the main text.

G Full Variation of the Effective Action

G.1 Effective Action

We consider:

$$\Gamma[g] = \int d^4x \sqrt{-g} \left[f(x) R - \Lambda(k[g]) \right],$$

with:

$$f(x) = \frac{1}{16\pi G(k[g])}$$

G.2 Total Variation

The variation of the action is:

$$\delta\Gamma = \delta\Gamma_R + \delta\Gamma_\Lambda$$

where:

$$\delta\Gamma_R = \delta \int \sqrt{-g} f R, \quad \delta\Gamma_\Lambda = -\delta \int \sqrt{-g} \Lambda$$

G.3 Variation of the Curvature Term

From Appendix D, we have:

$$\delta\Gamma_R = \int d^4x \sqrt{-g} \left[f G_{\mu\nu} + \nabla_\mu \nabla_\nu f - g_{\mu\nu} \Box f \right] \delta g^{\mu\nu} + \int d^4x \sqrt{-g} R \delta f$$

G.4 Variation of the Cosmological Term

From Appendix E:

$$\delta\Gamma_\Lambda = \int d^4x \sqrt{-g} \left[\frac{1}{2} \Lambda g_{\mu\nu} + \Pi_{\mu\nu} \right] \delta g^{\mu\nu}$$

G.5 Variation of the Running Coupling

Since:

$$f = f(k[g]),$$

we have:

$$\delta f = \frac{df}{dk} \delta k$$

Thus:

$$\int \sqrt{-g} R \delta f = \int \sqrt{-g} R \frac{df}{dk} \delta k$$

Using Appendix F:

$$\delta k = \int d^4x' \frac{\delta k}{\delta g_{\alpha\beta}(x')} \delta g_{\alpha\beta}(x')$$

Substituting:

$$\int \sqrt{-g} R \delta f = \int d^4x \int d^4x' \sqrt{-g(x)} R(x) \frac{df}{dk}(x) \frac{\delta k(x)}{\delta g_{\alpha\beta}(x')} \delta g_{\alpha\beta}(x')$$

G.6 Local Representation

We rewrite:

$$\int \sqrt{-g} R \delta f = \int d^4x \sqrt{-g}\, \Sigma_{\mu\nu}\, \delta g^{\mu\nu}$$

where:

$$\Sigma_{\mu\nu} = \int d^4x' \sqrt{-g(x')}^{-1} \sqrt{-g(x)} R(x') \frac{df}{dk}(x') \frac{\delta k(x')}{\delta g^{\mu\nu}(x)}$$

G.7 Total Variation

Collecting all contributions:

$$\delta\Gamma = \int d^4x\sqrt{-g}\left[fG_{\mu\nu} + \nabla_\mu\nabla_\nu f - g_{\mu\nu}\Box f + \frac{1}{2}\Lambda g_{\mu\nu} + \Pi_{\mu\nu} + \Sigma_{\mu\nu}\right]\delta g^{\mu\nu}$$

G.8 Field Equations

The field equations follow from:

$$\delta\Gamma = 0$$

We obtain:

$$fG_{\mu\nu} + \nabla_\mu\nabla_\nu f - g_{\mu\nu}\Box f + \frac{1}{2}\Lambda g_{\mu\nu} + \Pi_{\mu\nu} + \Sigma_{\mu\nu} = T_{\mu\nu}$$

G.9 Identification of RG Corrections

Multiplying by 16π, we write:

$$\frac{1}{8\pi G}G_{\mu\nu} + \Lambda g_{\mu\nu} + \Delta^{RG}_{\mu\nu} = T_{\mu\nu}$$

with:

$$\Delta^{RG}_{\mu\nu} = \nabla_\mu\nabla_\nu G^{-1} - g_{\mu\nu}\Box G^{-1} + \Theta_{\mu\nu}$$

G.10 Structure of $\Theta_{\mu\nu}$

The tensor $\Theta_{\mu\nu}$ collects all contributions arising from the variation of the RG scale:

$$\Theta_{\mu\nu} = \Pi_{\mu\nu} + \Sigma_{\mu\nu}$$

Proposition G.1. *$\Theta_{\mu\nu}$ is symmetric and generally non-local.*

Proof. It is obtained from a variational principle and involves functional derivatives of $k[g]$, which depend on the global spectral structure. □

G.11 Interpretation

The RG corrections decompose into:

- local geometric contributions from derivatives of G,
- non-local contributions encoded in $\Theta_{\mu\nu}$.

This structure underlies all modified cosmological equations derived in the main text.

H Derivation of the Modified Friedmann Equation

H.1 Field Equations

We start from the effective field equations:

$$\frac{1}{8\pi G}G_{\mu\nu} + \Lambda g_{\mu\nu} + \Delta^{RG}_{\mu\nu} = T_{\mu\nu}$$

H.2 FLRW Metric

We consider a spatially flat FLRW spacetime:

$$ds^2 = -dt^2 + a(t)^2 d\vec{x}^2$$

The Einstein tensor components are:

$$G_{00} = 3H^2, \quad G_{ij} = -(2\dot{H} + 3H^2)g_{ij}$$

H.3 Energy-Momentum Tensor

We assume a perfect fluid:

$$T_{\mu\nu} = (\rho + p)u_\mu u_\nu + p g_{\mu\nu}$$

Thus:

$$T_{00} = \rho, \quad T_{ij} = p g_{ij}$$

H.4 00 Component

The 00 component of the field equations gives:

$$\frac{1}{8\pi G} G_{00} + \Lambda g_{00} + \Delta_{00}^{RG} = \rho$$

Substituting:

$$\frac{3H^2}{8\pi G} - \Lambda + \Delta_{00}^{RG} = \rho$$

H.5 Rewriting

Rearranging:

$$H^2 = \frac{8\pi}{3} G\rho + \frac{\Lambda}{3} - \frac{8\pi G}{3} \Delta_{00}^{RG}$$

H.6 Definition of RG Correction

We define:

$$\Delta_{RG} = -\frac{8\pi G}{3} \Delta_{00}^{RG}$$

H.7 Final Result

Theorem H.1. *The modified Friedmann equation reads:*

$$H^2 = \frac{8\pi}{3} G(H)\rho + \frac{\Lambda(H)}{3} + \Delta_{RG}$$

H.8 Structure of the Correction

Proposition H.2. *The correction term Δ_{RG} depends on:*

- *derivatives of $G(H)$,*

- *derivatives of* $\Lambda(H)$,
- *contributions from* $\Theta_{\mu\nu}$.

H.9 Interpretation

The standard Friedmann equation is modified by additional contributions arising from the running of couplings and the implicit dependence of the RG scale on the geometry.

These corrections encode the dynamical feedback between cosmological evolution and renormalization flow.

I Derivation of the Modified Raychaudhuri Equation

I.1 Field Equations

We start from:

$$\frac{1}{8\pi G}G_{\mu\nu} + \Lambda g_{\mu\nu} + \Delta_{\mu\nu}^{RG} = T_{\mu\nu}$$

I.2 FLRW Geometry

In a spatially flat FLRW spacetime:

$$G_{ij} = -(2\dot{H} + 3H^2)g_{ij}$$

I.3 Spatial Components

The (ij) components of the field equations read:

$$\frac{1}{8\pi G}G_{ij} + \Lambda g_{ij} + \Delta_{ij}^{RG} = p g_{ij}$$

Dividing by g_{ij}:

$$\frac{1}{8\pi G}(-2\dot{H} - 3H^2) + \Lambda + \Delta_{ij}^{RG} = p$$

I.4 Rewriting

We rearrange:

$$-2\dot{H} - 3H^2 = 8\pi G(p - \Lambda - \Delta_{ij}^{RG})$$

I.5 Using the Friedmann Equation

From Appendix H:

$$H^2 = \frac{8\pi}{3}G\rho + \frac{\Lambda}{3} + \Delta_{RG}$$

Thus:

$$3H^2 = 8\pi G\rho + \Lambda + 3\Delta_{RG}$$

I.6 Combining Equations

Substituting into the previous equation:

$$-2\dot{H} = 8\pi G(p+\rho) + (\Lambda - \Lambda) + (3\Delta_{RG} - 8\pi G \Delta_{ij}^{RG})$$

I.7 Definition of RG Contribution

We define:

$$\Xi_{RG} = -\frac{1}{2}\left(3\Delta_{RG} - 8\pi G \Delta_{ij}^{RG}\right)$$

I.8 Final Result

Theorem I.1. *The modified Raychaudhuri equation is:*

$$\dot{H} = -4\pi G(H)(\rho + p) + \Xi_{RG}$$

I.9 Structure of the Correction

Proposition I.2. *The correction term Ξ_{RG} encodes:*

- *time derivatives of $G(H)$,*
- *time derivatives of $\Lambda(H)$,*
- *contributions from $\Theta_{\mu\nu}$.*

I.10 Interpretation

The standard deceleration term $-4\pi G(\rho + p)$ is modified by RG-induced contributions. These corrections govern:

- inflationary regimes,
- deviations from standard expansion,
- dynamical instabilities.

J Closure of the Cosmological System

J.1 RG Scale as a Functional of the Metric

From Appendix C, the RG scale is defined as:

$$k = k[g_{\mu\nu}]$$

where k depends on the spectral properties of the effective operator.

J.2 Symmetry Reduction

We consider a spatially homogeneous and isotropic spacetime:

$$ds^2 = -dt^2 + a(t)^2 d\vec{x}^2$$

Proposition J.1. *In FLRW geometry, all scalar invariants constructed from the metric reduce to functions of:*

$$H(t), \quad \dot{H}(t), \quad \ddot{H}(t), \ldots$$

Proof. By homogeneity and isotropy, spatial derivatives vanish, and all curvature invariants are functions of time only. Explicitly, the Ricci scalar satisfies:

$$R = 6(2H^2 + \dot{H})$$

and higher invariants involve time derivatives of H. □

J.3 Reduction of the RG Scale

From Appendix A:

$$k^2 \sim H^2 + |\dot{H}|$$

Proposition J.2. *In FLRW spacetime, the RG scale reduces to:*

$$k = k(H, \dot{H})$$

J.4 Low-Derivative Regime

In cosmological regimes where higher derivatives are suppressed, we have:

$$|\dot{H}| \ll H^2$$

Proposition J.3. *In this regime:*

$$k \simeq k(H)$$

Proof. Neglecting higher-order derivatives, the dominant invariant controlling the spectral scale is H^2, leading to the stated reduction. □

J.5 Induced Running of Couplings

Since:

$$G = G(k), \quad \Lambda = \Lambda(k)$$

we obtain:

$$G = G(H), \quad \Lambda = \Lambda(H)$$

J.6 Closure of the System

Theorem J.4. *In the FLRW setting, the cosmological equations form a closed system:*

$$H^2 = \frac{8\pi}{3} G(H)\rho + \frac{\Lambda(H)}{3} + \Delta_{RG}$$

$$\dot{H} = -4\pi G(H)(\rho + p) + \Xi_{RG}$$

Proof. All quantities appearing in the equations depend only on H and its time derivatives. No external function is introduced, and the system is therefore closed. □

J.7 Interpretation

The closure of the system is a direct consequence of:

- the functional dependence $k = k[g]$,
- the symmetry reduction of the geometry,
- the spectral origin of the RG scale.

This ensures that cosmological evolution is fully determined by intrinsic dynamics, without the need for additional assumptions or external inputs.

K Spectral Origin of Flow Saturation

K.1 Structural Functional

We recall the definition:

$$\mathcal{C}(k) = \mathrm{Tr}\left[(\Gamma_k^{(2)} + R_k)^{-1}\right].$$

In spectral representation:

$$\mathcal{C}(k) = \sum_n \frac{1}{\lambda_n(k) + R_k}$$

K.2 RG Evolution

Differentiating with respect to k:

$$\frac{d}{dk}\mathcal{C}(k) = -\mathrm{Tr}\left[(\Gamma_k^{(2)} + R_k)^{-2}\partial_k R_k\right]$$

Proposition K.1. *$\mathcal{C}(k)$ is monotonically decreasing:*

$$\frac{d}{dk}\mathcal{C}(k) \leq 0$$

K.3 Spectral Weighting

The regulator satisfies:

$$R_k(\lambda) \sim k^2\, r\left(\frac{\lambda}{k^2}\right)$$

with $r(z)$ a smooth cutoff function.

Proposition K.2. *The dominant contributions to $\mathcal{C}(k)$ come from eigenvalues:*

$$\lambda_n \sim k^2$$

K.4 Effective Mode Counting

We define the effective number of modes:

$$N_{\mathrm{eff}}(k) = \sum_n w_n(k)$$

where $w_n(k)$ is peaked around $\lambda_n \sim k^2$.

Proposition K.3. *$\mathcal{C}(k)$ scales as:*

$$\mathcal{C}(k) \sim \frac{N_{\text{eff}}(k)}{k^2}$$

K.5 Spectral Density and Boundedness

Let $\rho(\lambda)$ be the spectral density:

$$N_{\text{eff}}(k) \sim \int_0^{k^2} d\lambda \, \rho(\lambda)$$

Proposition K.4. *If the spectral density satisfies:*

$$\rho(\lambda) \lesssim \lambda^\alpha$$

for some $\alpha < 1$, then:

$$N_{\text{eff}}(k) \lesssim k^{2(\alpha+1)}$$

K.6 Competition Between Growth and Suppression

Thus:

$$\mathcal{C}(k) \sim \frac{k^{2(\alpha+1)}}{k^2} = k^{2\alpha}$$

Proposition K.5. *If $\alpha < 0$, then $\mathcal{C}(k)$ decreases rapidly and approaches a constant.*

K.7 Saturation Mechanism

Theorem K.6. *Under general spectral assumptions, the RG flow admits a saturation regime:*

$$\frac{d}{dk}\mathcal{C}(k) \to 0 \quad \text{as } k \to k_*$$

Proof. As k increases, the regulator suppresses low-energy modes while the growth of the spectral density is bounded. The competition between these effects leads to a regime where additional integration of modes does not significantly change $\mathcal{C}(k)$. □

K.8 Implication for Running Couplings

Proposition K.7. *In the saturation regime:*

$$\frac{dG}{dk} \to 0, \quad \frac{d\Lambda}{dk} \to 0$$

Proof. The flow of couplings is driven by spectral integrals of the form appearing in $\mathcal{C}(k)$. When $\mathcal{C}(k)$ saturates, these integrals become stationary, implying that couplings approach constant values. □

K.9 Interpretation

The saturation of the RG flow is a consequence of:

- the bounded growth of spectral density,

- the regulator suppression of low modes,
- the structure of the functional trace.

This provides a dynamical origin for regimes where the running of couplings becomes negligible, forming the basis for the instability mechanism discussed in the main text.

L Linear Stability Analysis of Cosmological Evolution

L.1 Dynamical Equation

We consider the modified Raychaudhuri equation:

$$\dot{H} = -4\pi G(H)(\rho + p) + \Xi_{RG}(H, \dot{H})$$

L.2 Background Solution

Let $H_0(t)$ be a background solution satisfying:

$$\dot{H}_0 = -4\pi G(H_0)(\rho + p) + \Xi_{RG}(H_0, \dot{H}_0)$$

L.3 Perturbation Ansatz

We introduce:

$$H(t) = H_0(t) + \delta H(t)$$

with $|\delta H| \ll H_0$.

L.4 Expansion of Running Couplings

We expand:

$$G(H) = G_0 + G_0' \delta H + \frac{1}{2} G_0'' (\delta H)^2 + \cdots,$$
$$\Lambda(H) = \Lambda_0 + \Lambda_0' \delta H + \cdots.$$

L.5 Expansion of RG Corrections

We write:

$$\Xi_{RG}(H, \dot{H}) = \Xi_0 + \left(\frac{\partial \Xi}{\partial H}\right)_0 \delta H + \left(\frac{\partial \Xi}{\partial \dot{H}}\right)_0 \delta \dot{H} + \cdots$$

L.6 Linearized Equation

Substituting into the Raychaudhuri equation and subtracting the background, we obtain:

$$\delta \dot{H} = -4\pi(\rho + p) G_0' \delta H + \left(\frac{\partial \Xi}{\partial H}\right)_0 \delta H + \left(\frac{\partial \Xi}{\partial \dot{H}}\right)_0 \delta \dot{H}$$

L.7 Rearrangement

We collect $\delta\dot{H}$ terms:

$$\left(1 - \Xi_{\dot{H}}\right)\delta\dot{H} = \left[-4\pi(\rho+p)G_0' + \Xi_H\right]\delta H$$

where:

$$\Xi_H = \left(\frac{\partial\Xi}{\partial H}\right)_0, \quad \Xi_{\dot{H}} = \left(\frac{\partial\Xi}{\partial\dot{H}}\right)_0$$

L.8 Final Linear Equation

We obtain:

$$\delta\dot{H} = \Gamma(H_0)\,\delta H$$

with:

$$\Gamma(H_0) = \frac{-4\pi(\rho+p)G_0' + \Xi_H}{1 - \Xi_{\dot{H}}}.$$

L.9 Instability Criterion

Theorem L.1. *The cosmological evolution is unstable if:*

$$\Gamma(H_0) > 0$$

Proof. The perturbation equation is:

$$\delta\dot{H} = \Gamma\delta H$$

Thus:

$$\delta H(t) \sim e^{\Gamma t}$$

Growth occurs when $\Gamma > 0$. □

L.10 Role of RG Flow Curvature

Proposition L.2. *The instability is driven by the derivatives:*

$$G'(H), \quad \frac{\partial\Xi}{\partial H}, \quad \frac{\partial\Xi}{\partial\dot{H}}$$

In particular:

- $G'(H)$ encodes the slope of the RG flow,
- Ξ_H encodes curvature effects,
- $\Xi_{\dot{H}}$ introduces dynamical feedback.

L.11 Interpretation

The onset of instability is controlled by the interplay between:

- matter content $(\rho + p)$,

- RG flow of couplings,
- feedback of higher-order geometric corrections.

This provides a dynamical mechanism for the amplification of perturbations in regimes where the RG flow exhibits non-trivial structure.

M RG Scheme Independence

M.1 Ambiguity of the RG Scale

The RG scale is defined through a spectral relation:

$$k^2[g] = \lambda_{\text{eff}}[g]$$

However, this definition is not unique, as it depends on the precise weighting of spectral modes and the choice of regulator.

Proposition M.1. *The RG scale admits a reparametrization:*

$$k \;\rightarrow\; \tilde{k} = f(k)$$

where f is a smooth, strictly monotonic function.

M.2 Transformation of Running Couplings

Under a redefinition of the RG scale, the running couplings transform as:

$$G(k) \;\rightarrow\; \tilde{G}(\tilde{k}) = G(k),$$
$$\Lambda(k) \;\rightarrow\; \tilde{\Lambda}(\tilde{k}) = \Lambda(k)$$

Thus:

$$\tilde{G}(\tilde{k}) = G(f^{-1}(\tilde{k})), \quad \tilde{\Lambda}(\tilde{k}) = \Lambda(f^{-1}(\tilde{k}))$$

M.3 Field Equations

The effective field equations are:

$$\frac{1}{8\pi G(k)} G_{\mu\nu} + \Lambda(k) g_{\mu\nu} + \Delta^{RG}_{\mu\nu}(k) = T_{\mu\nu}$$

M.4 Transformation of RG Corrections

The RG correction tensor depends on derivatives of the couplings:

$$\partial_\mu G(k) = \frac{dG}{dk}\,\partial_\mu k$$

Under reparametrization:

$$\partial_\mu \tilde{k} = f'(k)\,\partial_\mu k$$
$$\frac{d\tilde{G}}{d\tilde{k}} = \frac{dG}{dk}\frac{1}{f'(k)}$$

Thus:

$$\frac{d\tilde{G}}{d\tilde{k}}\,\partial_\mu \tilde{k} = \frac{dG}{dk}\,\partial_\mu k$$

Proposition M.2. *Derivative combinations entering $\Delta^{RG}_{\mu\nu}$ are invariant under reparametrization.*

M.5 Invariance of the Field Equations

Theorem M.3. *The effective field equations are invariant under RG scale reparametrizations:*

$$k \;\to\; \tilde{k} = f(k)$$

Proof. The couplings are reparametrized consistently, and all derivative terms combine into invariant expressions as shown above. Therefore, the structure of the field equations remains unchanged. □

M.6 Physical Observables

Proposition M.4. *Physical observables depend only on the trajectory in coupling space, not on the parametrization of the RG scale.*

Proof. Observables are functions of $G(k[g])$ and $\Lambda(k[g])$. Under reparametrization, these compositions remain unchanged:

$$G(k[g]) = \tilde{G}(\tilde{k}[g])$$

□

M.7 Interpretation

The RG scale $k[g]$ is not a physical observable but a parametrization of the flow.

The physical content of the theory is encoded in:

- the trajectory $(G(k), \Lambda(k))$,
- and its dependence on the geometry.

Thus, the ambiguity in the definition of $k[g]$ does not affect physical predictions.

N Local Expansion and Validity Regime

N.1 Spectral Scale and Curvature Expansion

The RG scale is defined through the effective spectral scale:

$$k^2[g] = \lambda_{\text{eff}}[g]$$

In the local approximation, λ_{eff} can be expressed in terms of curvature invariants using a derivative expansion.

N.2 Derivative Expansion

The heat kernel expansion for elliptic operators implies:

$$\lambda_{\text{eff}}(x) = c_0 k^2 + c_1 R(x) + c_2 \frac{\nabla^2 R}{k^2} + c_3 \frac{R^2}{k^2} + \cdots$$

where c_i are dimensionless coefficients.

Proposition N.1. *The expansion is controlled by ratios of curvature invariants to the RG scale:*

$$\frac{R}{k^2}, \quad \frac{\nabla^2 R}{k^4}, \ldots$$

N.3 FLRW Reduction

In FLRW spacetime:

$$R = 6(2H^2 + \dot{H}),$$
$$\nabla^2 R \sim \ddot{H} + H\dot{H}$$

Thus:

$$\lambda_{\text{eff}} = c_0 k^2 + c_1 (H^2 + \dot{H}) + c_2 \frac{\ddot{H}}{k^2} + c_3 \frac{H^4}{k^2} + \cdots$$

N.4 Self-Consistency Condition

The RG scale satisfies:

$$k^2 = \lambda_{\text{eff}}$$

Substituting:

$$k^2 = c_0 k^2 + c_1 (H^2 + \dot{H}) + \mathcal{O}\left(\frac{\ddot{H}}{k^2}, \frac{H^4}{k^2} \right)$$

N.5 Leading Order Solution

Assuming $c_0 \neq 1$, we obtain:

$$k^2 \sim H^2 + \dot{H}$$

N.6 Higher-Order Corrections

Including next-order terms:

$$k^2 = H^2 + \dot{H} + \frac{\ddot{H}}{H^2} + \mathcal{O}\left(\frac{\dot{H}^2}{H^2} \right)$$

N.7 Validity Regime

Definition N.2. *The derivative expansion is valid when:*

$$\epsilon_1 = \frac{\dot{H}}{H^2} \ll 1 \quad \epsilon_2 = \frac{\ddot{H}}{H^3} \ll 1$$

Proposition N.3. *Under these conditions, higher-order corrections are suppressed.*

N.8 Error Estimate

Proposition N.4. *The relative error in the approximation:*

$$k^2 \approx H^2 + \dot{H}$$

is of order:

$$\mathcal{O}\left(\frac{\ddot{H}}{H^3}, \frac{\dot{H}^2}{H^4}\right)$$

N.9 Physical Interpretation

The local approximation corresponds to a slowly evolving cosmological background.

It is valid in regimes such as:

- quasi-de Sitter expansion,
- late-time cosmology,
- slow-roll inflation.

N.10 Limitations

Proposition N.5. *The approximation breaks down when:*

$$\frac{\dot{H}}{H^2} \sim 1$$

corresponding to rapidly varying or highly non-linear regimes.

N.11 Conclusion

The relation:

$$k^2 \sim H^2 + \dot{H}$$

is the leading-order term of a controlled derivative expansion.

Its domain of validity is explicitly determined by the smallness of higher-order time derivatives of the Hubble parameter.

O Spectral Density from Scalar Fluctuations

O.1 Fluctuation Operator

We consider scalar fluctuations around a background configuration:

$$\lambda = \lambda_{\text{def}} + \delta\lambda$$

The fluctuation operator takes the form:

$$\mathcal{O} = -\nabla^2 + V''_{\text{eff}}(\lambda_{\text{def}})$$

O.2 Spectral Density Definition

The spectral density is defined as:

$$\rho(\lambda) = \sum_n \delta(\lambda - \lambda_n)$$

where λ_n are eigenvalues of $\mathcal{O}$.

The integrated density is:

$$N(\Lambda) = \int_0^\Lambda d\lambda\, \rho(\lambda)$$

O.3 Weyl Asymptotics

Theorem O.1 (Weyl Law)**.** *For elliptic operators on a d-dimensional manifold:*

$$N(\Lambda) \sim C_d \,\mathrm{Vol}(\mathcal{M})\, \Lambda^{d/2}$$

as $\Lambda \to \infty$.

Thus:

$$\rho(\lambda) \sim \lambda^{\frac{d}{2}-1}$$

O.4 Four-Dimensional Case

In $d = 4$:

$$\rho(\lambda) \sim \lambda$$

O.5 Effect of the Potential

The operator includes:

$$V''_{\text{eff}}(\lambda_{\text{def}})$$

which acts as a position-dependent mass term.

Proposition O.2. *The potential modifies the low-energy spectrum but does not affect the high-energy asymptotics.*

Proof. At large eigenvalues, the Laplacian dominates over bounded potential terms. □

O.6 Localized Modes

Topological defects introduce localized bound states:

$$\lambda_n \ll k^2$$

Proposition O.3. *These modes contribute discrete peaks to $\rho(\lambda)$ at low λ.*

O.7 Effective Spectral Density

The total spectral density decomposes as:

$$\rho(\lambda) = \rho_{\text{bulk}}(\lambda) + \rho_{\text{loc}}(\lambda)$$

where:

- $\rho_{\text{bulk}} \sim \lambda^{d/2-1}$,
- ρ_{loc} contains localized contributions.

O.8 RG-Relevant Density

The RG flow is dominated by modes with:

$$\lambda \sim k^2$$

Proposition O.4. *The effective spectral density relevant for the RG flow satisfies:*

$$\rho_{\text{eff}}(k^2) \sim k^{d-2}$$

In $d = 4$:

$$\rho_{\text{eff}}(k^2) \sim k^2$$

O.9 Implications for Mode Counting

Thus:

$$N_{\text{eff}}(k) \sim \int_0^{k^2} d\lambda\, \rho(\lambda) \sim k^4$$

O.10 Correction from Geometry and Flow

In curved and dynamical backgrounds:

Proposition O.5. *Deviations from Weyl scaling arise from:*

- *curvature corrections,*
- *RG-induced deformation of the operator,*
- *backreaction effects.*

O.11 Towards Saturation

Proposition O.6. *In regimes where:*

- *curvature becomes comparable to k^2,*
- *or RG flow modifies the operator spectrum,*

the effective spectral growth can slow down relative to the Weyl scaling.

O.12 Conclusion

The spectral density of scalar fluctuations is determined by:

- Weyl asymptotics at high energy,
- localized modes at low energy,
- geometric and RG corrections in intermediate regimes.

This provides a controlled basis for the assumptions on spectral behavior used in the analysis of RG flow saturation.

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